Potential Research Topics:

* **Martingale**
  + Modelling “fair game” scenarios where given a sequence of random variables and then growing information which represents “everything known up to time n” where E[Xn+1 | Fn] = Xn
  + Designing martingale tests in models can be helpful to diagnose if your model is drifting towards a bias over time
  + Interesting subtopics:
    - Bandit
    - SGD
    - TD-learning
    - Localized Martingales
    - Super/Submartingales
* **Renewal Process**
  + System that renews itself at random time intervals e.g. replacing a machine part that wears out
  + Poisson Explanation:
    - The probability of an event occurring in the next instant is always the same
  + Renewal Process expansion:
    - Generalizes the Poission process, modelling events that “reset the system” but allow for non-exponential inter-arrival times
    - Not memoryless, past events influence future behavior
  + Renewal Function
    - N(t) is the number of renewals that have occurred by time t.
    - Inter-arrival times Ti are i.i.d with distribution function F(t) and mean μ=E[T]
      * Generally, F(t) is harder to compute due to it being an arbitrary function.
  + Elementary Renewal Theorem (ERT)
    - Long-run behavior of the renewal process:
      * Over long-time horizon, rate of renewals stabilizes at 1 / , the reciprocal of the mean inter-arrival time
      * Even if distribution is not exponential, average rate will hold in the long run
        + Making it useful for knowing average number of inter-arrival events per time period
  + Delayed Renewal Process
    - First inter-arrival time T1 has a different distribution form the rest…
    - The remaining inter-arrival times are i.i.d. from a common distribution F
    - Formal Definition:
      * G(t): CDF of first inter-arrival time T1
      * F(t): CDF of all other inter-arrival times Ti, for i >= 2
    - Examples: Warranties, Medical trials
    - Renewal Function for Delayed Process
      * MD(t): expected number of renewals up to time t in a delayed renewal process
      * M(t): the standard renewal function with i.i.d. inter-arrival F
    - Then
      * MD(t) = G(t) +
      * Shows that the delayed renewal function depends on the distribution of the first interval and the standard renewal behavior afterwards
    - Still holds that the asymptotic rate is 1 / , short term behavior differs drastically
  + Renewal Reward Theorem (RRT)
    - Each renewal or cycle earns a reward, then the average reward rate stabilizes over time
    - Formal Statement:
      * Ti: inter-arrival times (i.i.d with E[T] = )
      * Ri: reward earned in the i-th cycle (i.i.d. with E[R] = η)
      * R(t) = : total reward by time t
      * (w/ probability 1 and in expectation)
        + On average, you get η reward per cycle and cycles occur at rate 1 / mu. So reward per unit time approaches η / mu
  + Inspection Paradox
    - When observing a system at a random time, you’re more likely to catch it in a longer-than-average interval, e.g. bus arrival times
    - T 🡪 Time interval, R 🡪 Residual Life (time from random observation to next event)
  + Residual Life Analysis
    - A(t): age (time since last renewal)
    - R(t): residual life (time until next renewal)
    - Then:
    - Limiting Distribution of Residual Life:
    - Density:
* **Gaussian/Brownian Motion**
  + Continuous-time stochastic process modelling random continuous movement
    - Can just be thought of as a continuous time martingale
  + Process B(t) is standard Brownian motion if:
    - B(0) = 0
    - Independent increments: For s < t, B(t) – B(s) is independent of the past.
    - Gaussian Increments: B(t) – B(s) ~ N(0, t-s)
    - Continuous Paths: function t 🡪 B(t) is continuous almost surely
  + Can be thought of as the continuous limit of a random walk
    - Taking steps left/right very rapidly w/ small step size
    - As time step shrinks to zero, converges to a stochastic process
  + Potential Applications and exploration into survival analysis